International Journal of Modern Physics C, Vol. 0, No. 0 (1997) 000–000


# LATTICE SIMULATION OF NUCLEAR MULTIFRAGMENTATION

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Received (received date)
Revised (revised date)

Motivated by the decade-long debate over the issue of criticality supposedly observed in nuclear multifragmentation, we propose a dynamical lattice model to simulate the phenomenon. Its Ising Hamiltonian mimics a short range attractive interaction which competes with a thermal-like dissipative process. The results here presented, generated through an event-by-event analysis, are in agreement with both experiment and those produced by a percolative (non dynamical) model.

## 1. Introduction

In the early 1980's, the experimental observation of power law-like distribution of fragment yields in nuclear reactions induced by high energy proton projectiles [1,2] initiated an ongoing controversy on the origins of this behavior. When a proton or a heavy ion collides with a heavy nucleus, the target can break into a number of nuclei with atomic number $Z > 2$ in addition to several nucleons and alpha particles. The number of heavy fragments induces a classification of these processes into three different regimes: *fission*, with two heavy fragments; *spallation*, yielding one heavy fragment and a few light ones; and *multifragmentation*, where the product includes no heavy fragments, but is composed of several light nuclei of varying sizes [3]. There is general agreement that the boundary between light and heavy fragments lies around $Z = 20$ [4]. The processes of fission and spallation are quite well understood, but such is not the same with multifragmentation. One of the reasons for this lack of knowledge stems from experimental difficulties - a detailed investigation of fragmentation requires coincident measurements of the multiple fragments formed. Most of the experiments reported in the literature measure little more than the inclusive mass yield of fragments - and so cannot make a clear distinction between those that correspond to each kind of phenomenon involved. Most of the exclusive experiments, on the other hand, are emulsion experiments with inverse kinematics [1], and suffer the drawback of poor statistics. The lack of reliable coincidence data has forced theoretical investigations to concentrate on the explanation of the mass yield curve.

For $Z \leq 20$, it has been pointed out that this curve is compatible with a power-law dependence for the cross-section $\sigma \sim Z^{-\tau}$, where $\tau \sim 2.2$ is practically independent of the beam energy and the exact composition of projectile and target. This parametrization of the mass yield curve, along with the measured value of the exponent, is precisely what is expected for the transition between liquid and gaseous phases in nuclear matter [4] - this transition takes its name from a parallel with the process of droplet formation in a similar transition for fluids [5]. This fact has been pointed out as a clear indication that multifragmentation is strongly related to that phase transition.

The currently accepted scenario for the fragmentation of nuclei begins with a quick isentropic expansion, as the target nucleus is hit [6]. The resulting compound nucleus has its density diminished in this process, and the fragmentation begins with a cracking of the system. In this way, primary fragments are formed, still highly excited. Deexcitation of these fragments is acompanied by the formation of secondary fragments [7]. During the whole process, the system has enough time for thermal equilibration; we have two different time-scales involved - namely, in this case, we have fast formation of cracks and slow thermalization -, one of the known conditions for the installation of a complex regime. The theoretical understanding of this process is very difficult, since we are in the presence of a true many-body problem, due to long range correlations between the clusters that are being formed.

Among the theoretical models that have been proposed to study this phenomenon ( for a brief review, see Ref.[8] ), by far the most popular one is a bond-percolation model on a cubic lattice [9]. It has been shown that, independently of the knowledge of the parameter that would control the approach to criticality, a number of successful comparisons could be made between predictions of this model and experimental results [10]. By studying the moments of the cluster size distributions on an event-by-event analysis, and assuming a scaling property for these distributions [11], one can show that a strong correlation between these moments must be present in the vicinity of the critical region. Nevertheless, the purely geometrical and static character of such a model brings about the need for one that could rely on some dynamical interaction between the nucleons participating in the process.

In the next section, we present a dynamical dissipative lattice model that responds to that need. Some details of its computer implementation are then discussed. The last section is dedicated to a presentation of our results, and to a comparison between them and those coming from the percolative model.

## 2. The Model

Initial versions of the model we present in this paper were used in the context of surface wetting [12] and of drop formation on the leaky faucet problem [13,14]. Its use in nuclear collisions was foreseen in the analysis of residual mass in the evaporation of hot nuclei [15]. The success obtained in that application encouraged us to the extension presented here.

The nucleus, in its initial configuration, is a dense cluster of occupied cells in an otherwise empty cubic lattice - using the terminology of magnetic systems in the context of the gas-lattice model, a cluster of spins up surrounded by spins down - subject to an Ising-type interaction given by the model Hamiltonian

$$\mathcal{H} = -J \sum_{<i,j>} S_i S_j, \tag{1}$$

where the summation extends to nearest and next-nearest neighbours and $S_{i,j} = \pm 1$. With this last prescription, we intend to mimic a surface tension, having in mind a liquid drop model for the nucleus. The initial excitation energy of this compound nucleus - target plus absorbed projectile - is associated to a temperature parameter, which will control the transition probability between two different configurations of the system. The nucleus is then subjected to a dissipative Metropolis dynamics [16], generating a Markovian chain of configurations. This dynamics involves:
(i) a random choice of an occupied site at the cluster perimeter, followed by a random choice of an unoccupied site, not contiguous to the first, also at the perimeter;
(ii) a double flip of the spins at these two sites, decided by the Metropolis rule - thus involving the temperature parameter already mentioned;
(iii) if the double flip occurs, it is verified if this configuration is still a connected set of spins up; if it is not, we have the formation of a primary fragment, whose mass is contabilized, and energy is dissipated, in the form of a decrease in temperature (intermittent, fast cracking energy dissipation);
(iv) in either case, an additional small decrement in temperature is promoted, simulating the emission of radiation by the system (continuous, slow thermalization).

The recently formed fragment is erased, and this dynamics continues until a low temperature is attained, when no more fragments would be formed.

The distribution of fragments thus obtained is characterized by its moments. If $n_i(s)$ is the number of fragments with mass $s$ obtained in the event $i$, its $k^{th}$ moment is computed by

$$M_i(k) = \sum_s s^k n_i(s).$$

In a normal thermodynamic system, these moments would diverge in the thermodynamic limit at the critical point, for $k > 1$, with critical exponents

$$M(k, \epsilon) \sim \epsilon^{-\mu_k},$$

where $\epsilon$ measures the distance to the critical point. If the distribution $n(s)$ has the scaling property [5,11]

$$n(s, \epsilon) \sim s^{-\tau} f(\epsilon s^{\sigma}),$$

where $\tau$ and $\sigma$ are two critical exponents, one can show that moments of different orders must be correlated. The relation between the exponents of moments divergence and those of the distribution is given by [10]

$$\mu_k = -(\tau - 1 - k)/\sigma.$$

Since we are working with intrinsically finite systems - the nuclei - the moments will remain finite, even for $k > 1$. The normal signature of critical behavior - the divergence of the moments of the distribution - is washed out by the finiteness of the system. This is one of the difficulties that faces theoretical work in small systems. Nevertheless one can use the correlation between moments of different orders to examine the surviving traces of critical behavior.

It is more natural to work with normalised moments

$$S_i(k) = M_i(k)/M_i(1)$$

instead of the regular ones. Then it can be shown [10] that

$$\log(S_i(3)) = \lambda_{3/2}\log(S_i(2))$$

and

$$\log(S_i(5)) = \lambda_{5/2}\log(S_i(2))$$

where, with the usual identification $\gamma = \mu_2$,

$$\lambda_{3/2} = 1 + 1/\sigma\gamma,$$

and

$$\lambda_{5/2} = 1 + 3/\sigma\gamma.$$

## 3. Computer Strategy

We will adopt the C programming language sintax [17] in this section, with some minor explanations. Our strategy follows two rules: storing data (spin states) bit by bit on computer words; and treating them almost exclusively by bitwise logical operators. Obviously, this saves computer memory by a factor of 32 (for 32 bit word computers). Better yet, computer time can be saved by a similar factor whenever parallel updating is possible; such is the case in some of the routines discussed below. Many general tricks designed to implement this strategy, similar to the particular ones used here, are discussed in detail in references [14,18].

Our model nucleus resides on a 32x32x32 cubic lattice. This lattice is mapped onto a vector L[$r$] ($r = 0, 1, 2 \ldots lastword$), that keeps the current nucleus shape, through the rule: element $(i, j, k)$ is represented by the $k^{th}$ bit of L[$j * 32 + i$] ($lastword$ is a mnemonic for $32 * 32 - 1$). The rationale behind this choice lies in the fact that, although a mathematical operation is needed whenever a particular cell must be accessed, that is exactly what is done at machine code level - so very little extra time is spent. On the other hand, most of the really time-consuming operations are done on a sequential basis; in these cases, storing the array as a single-index vector avoids the need to go through multiple accesses by pointers, which is the way arrays are usually implemented on computers.

As an example of all that has just been said, we write down the algoritm used to determine the inner nucleus boundary. This boundary is stored on array I[$r$] by

```
for(r = 0; r <= lastword; r++) {
    I[r] = (~L[r]<<1) ¦ (~L[r]>>1) ¦ ~L[r-1] ¦ ~L[r+1] ¦ ~L[r+32] ¦ ~L[r-32];
/* first neighbors */
    I[r] = I[r] ¦ ~L[r - 1 - 32] ¦ ~L[r - 1 + 32] ¦ ~L[r + 1 - 32] ¦ ~L[r + 1 + 32] ¦
(~L[r-1]<<1) ¦ (~L[r-1]>>1) ¦ (~L[r+1]<<1 ¦ (~L[r+1]>>1) ¦ (~L[r-32] <<1)
¦ (~L[r - 32]>>1) ¦ (~L[r + 32]<<1) ¦ (~L[r + 32]>>1); /* and second neighbors
*/
    I[r] = I[r] & L[r]; }
```

where ~, ¦ and & represent respectively the bitwise logical operations NOT, OR and AND. A similar procedure gives the outer drop boundary. A pair of non-adjacent positions are randomly selected, one from each of these boundaries, as candidates for a mass-conserving, Kawasaki-like updating. The energy variation involved in this exchange of positions is also calculated through the use of bitwise logical operations.

The determination of the connectedness of the nucleus after the spin updating is a dificult problem, due to its non-locality. There is no clear shortcut solution in the literature for three dimensions, and here resides the most time-consuming algorithm in the program. We chose the so-called burning algorithm [19], designed to determine connected clusters of lattice sites sharing some property. A computer implementation for a similar problem in two dimensions was published in Ref.[14].

## 4. The Results

For a close examination of the correlation between normalised moments of different orders, we made log-log plots of $S(3) \times S(2)$ and $S(5) \times S(2)$, following the suggestions originally made by Campi [10]. In fig. 1 and 2 we show the evolution of these correlations with the initial excitation energy of the system. It is clear from these graphs that the model reproduces the strong correlation observed experimentally, as also did the percolation model. The points making an apparent arc of circle near the origin for low excitation energies in both instances are themselves also present in the experiments, and are probably related to very 'gentle' events, yielding mostly fragments of small mass ($< 4$). The measured slopes $\lambda_{3/2} = 2.28 \pm 0.01$ and $\lambda_{5/2} = 4.75 \pm 0.04$ agree with both experiment and percolation model [10].

Observation of the data gives an indication that our model is probably more 'selective' in the energy involved in the reaction then the experiments they are being compared with - the data used was from inclusive collisions [1]. A selection of the initial temperature parameter can bring the system closer to the critical region. This makes it a candidate for modelling the new exclusive experiments now being made.

The determination of the proximity to the critical region made use of the so called 'Campi scatter plot' (fig.3). This is a log-log plot of the greatest fragment

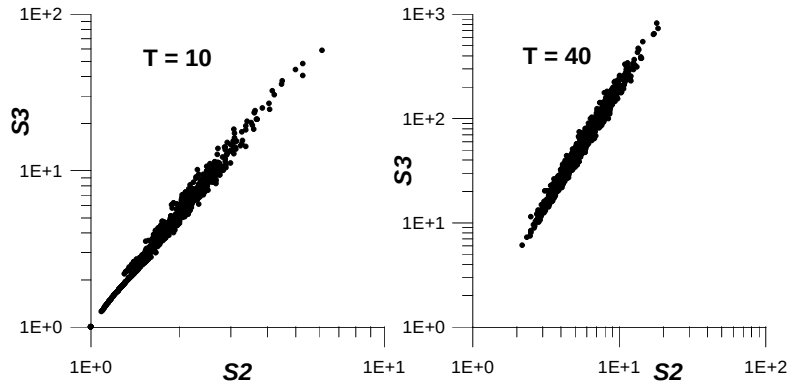


Fig. 1. Plot of $\log(S_3) \times \log(S_2)$. The system used in simulations is a 6x6x6 cubic cluster - total number of "nucleons" is 216, permitting close comparisons with the results quoted by Ref.[10].

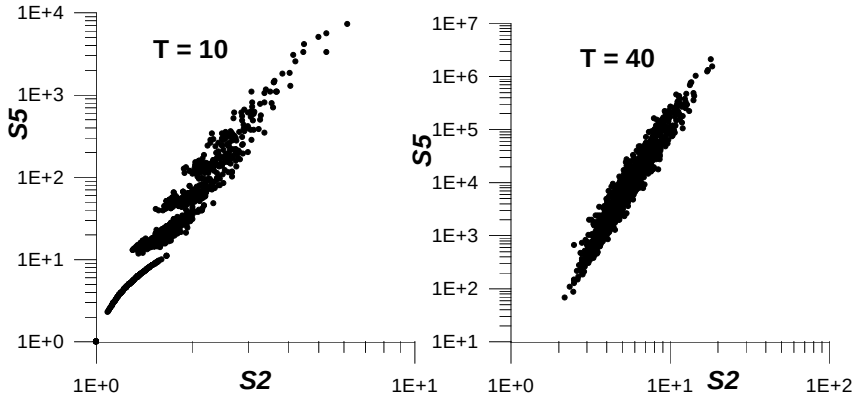


Fig. 2. Plot of $\log(S_5) \times \log(S_2)$. The points closer to the origin on the graph on the left correspond to highly undercritical events, as explained in the text.

in each event against the second normalized moment, averaged over events with the same $M(1)$. In this plot one can identify three different regions [20]: a region with negative slope, corresponding to events with small energy - or *undercritical* -, a region with positive slope, or *overcritical* and a region where there is a great dispersion in the second moment. We understand that this is the critical regime we are looking for. In fig.3 we show the evolution of this plot with the increase in the temperature parameter. The simulation with $T = 40$ was chosen as a representative of the experimental situation, and determined the temperature where the slopes $\lambda_{3/2}$ and $\lambda_{5/2}$ were measured. The measured slope of the scatter plot in the undercritical region, although very inaccurate, is also coherent with the result of the percolative model.

In summary, we showed that a dissipative dynamic model can reproduce much

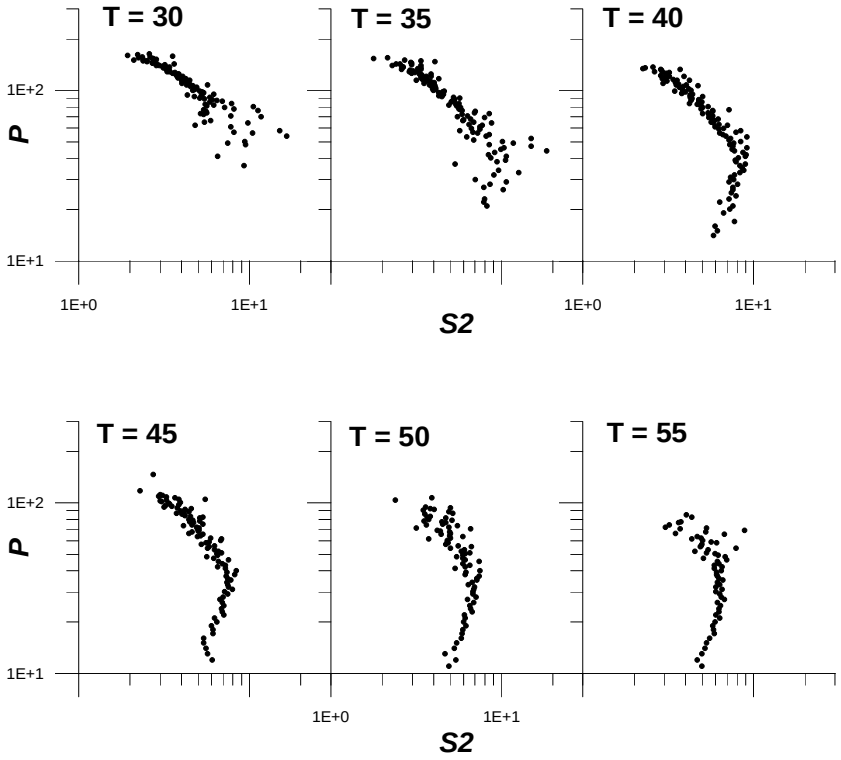


Fig. 3. Evolution of Campi's Scatter Plot with the initial value of the temperature parameter. As this value is increased, the system goes more and more into the critical regime, loosely associated with events represented by points at the intersection of the negative and positive slopes portions of the graph.

of the observed behaviour of the distribution of fragments produced in nuclear reactions induced by collisions with high energy protons or heavy ions. These results allow us to conclude that a few model dynamical ingredients, namely a short range nucleon-nucleon attraction, fast energy dissipation during each crack formation and continuous, slow thermalization are enough: a detailed description of the microscopic interactions is not needed. A complete characterisation of the fragment distribution generated through the use of this model is now in progress.

## Acknowledgements

We would like to thank S.M. de Oliveira, T.J.P.Penna, and A. Delfino for helpfull discussions. This work was partially suported by Brazilian agencies CNPq, FINEP, CAPES and FAPERJ.

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